Van Hiles Theory of geometric Thinking

Introduction

- Pierre van Hiele and his wife Dina van Hiele-Geldof were Dutch researchers and teachers.
- They had personal experience with difficulties which their students had in learning geometry. Therefore, they dealt with these problems in detail.
- The theory originated in their theses at the University of Utrecht in 1957. Pierre van Hiele devoted his lifetime to their theory, Dina died shortly after completing her thesis.
- Research based on the theory was carried out in the Soviet Union in the 1960s. Using its results, a very successful new geometry curriculum was designed in the Soviet Union. American researchers did several large studies on the van Hiele theory in the late 1970s [Usiskin, 1982 and Senk, 1985]. These studies influenced American NCTM Standards and Common Core State Standards.

Van Hiele theory

- The theory has three aspects:
  - the existence of levels,
  - the properties of the levels, and
  - the progress from one level to the next level.

Van Hiele levels

- According to the theory, there are five levels of thinking or understanding in geometry:
  - Level 0 Visualization
  - Level 1 Analysis
  - Level 2 Abstraction
  - Level 3 Deduction
  - Level 4 Rigor

Level 0. Visualization - Basic Visualization or recognization

At this level pupils use visual perception and nonverbal thinking. They recognize geometric figures by their shape as "a whole" and compare the figures with their prototypes or everyday things ("it looks like door"), categorize them ("it is / it is not a…"). They use simple language. They do not identify the properties of geometric figures.

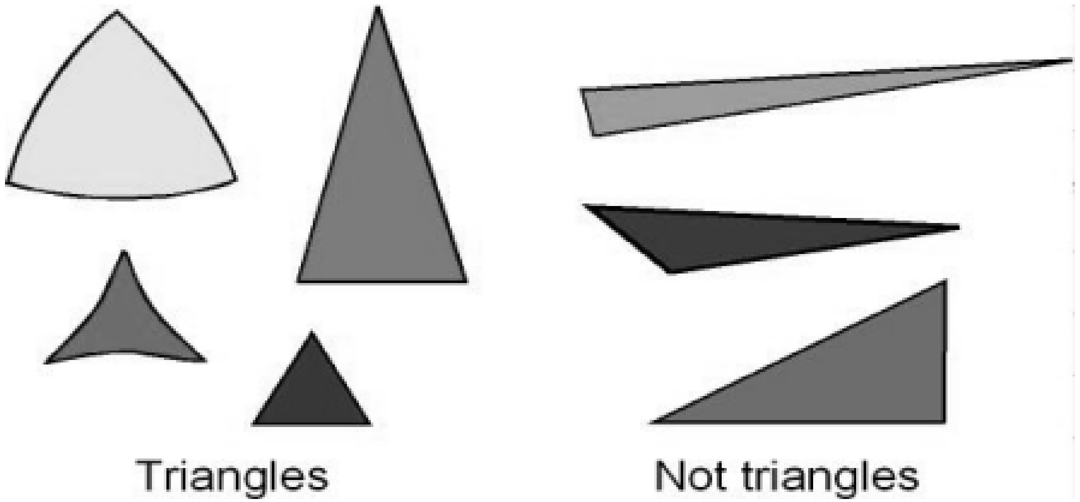

Figure 1. Children at Level 0 categorize triangles.

Level 1. Analysis (Description)

- At this level pupils (students) start analyzing and naming properties of geometric figures.
- They do not see relationships between properties, they think all properties are important (= there is no difference between necessary and sufficient properties).
- They do not see a need for proof of facts discovered empirically.
- They can measure, fold and cut paper, use geometric software etc

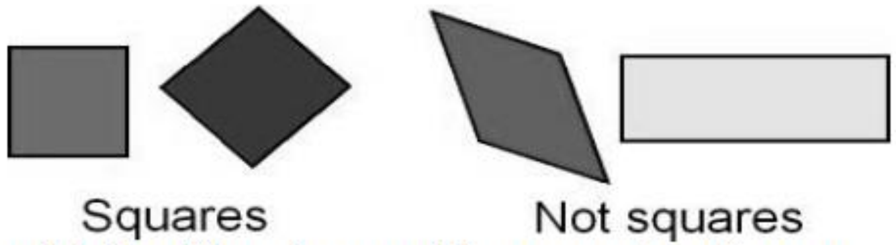

Figure 2. Children at Level 1 identify only one of the properties of squares.

Level 2 Abstraction (Informal deduction or Ordering or Relational)

At this level pupils or students perceive relationships between properties and figures. They create meaningful definitions. They are able to give simple arguments to justify their reasoning. They can draw logical maps and diagrams. They use sketches, grid paper, geometric SW.

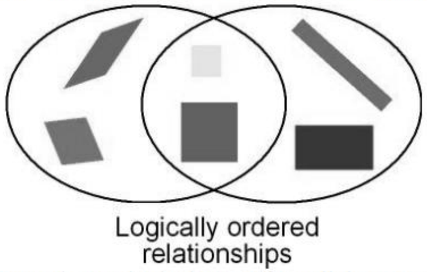

Figure 3. Children at Level 2 can draw a logical map of parallelograms.

Pierre van Hiele wrote: “My experience as a teacher of geometry convinces me that all too often, students have not yet achieved this level of informal deduction. Consequently, they are not successful in their study of the kind of geometry that Euclid created, which involves formal deduction.”

Level 3 Deduction (Formal deduction)

- At this level students can give deductive geometric proofs. They are able to differentiate between necessary and sufficient conditions. They identify which properties are implied by others. They understand the role of definitions, theorems, axioms and proofs.

Level 4 Rigor

At this level students understand the way how mathematical systems are established.

They are able to use all types of proofs.

They comprehend Euclidean and non-Euclidean geometry.

They are able to describe the effect of adding or removing an axiom on a given geometric system.

Properties of levels

- The levels have five important characteristics:
- **Fixed sequence (order):** A student cannot be at level N without having gone through level (N−1). Therefore, the student must go through the levels in order.
- **Adjacency:** At each level, what was intrinsic (basic) in the preceding level becomes extrinsic (related) in the current level.
- **Distinction:** Each level has its own linguistic symbols and its own network of relationships connecting those symbols. The meaning of a linguistic symbol is more than its explicit definition; it includes the experiences which the speaker associates with the given symbol. What may be "correct" at one level is not necessarily correct at another level.
- **Separation:** Two persons at different levels cannot understand each other. The teacher speaks a different "language" to the student at a lower level. The van Hieles thought this property was one of the main reasons for failure in geometry. (fail to make connection)
- **Attainment:** The learning process leading to complete understanding at the next level has five phases – information, guided orientation, explanation, free orientation, integration, which are approximately not strictly sequential.

Five phases of the learning process

Van Hieles believed that cognitive progress in geometry can be accelerated by instruction.

The progress from one level to the next one is more dependent upon instruction than on age or maturity.

They gave clear explanations of how the teacher should proceed to guide students from one level to the next. However, this process takes tens of hours.

Five phases of the learning process

- **Information or Inquiry** Students get the material and start discovering its structure. The teacher holds a conversation with the pupils, in well-known language symbols, in which the context he wants to use becomes clear. (A teacher might say: “This is a rhombus. Construct some more rhombi on your paper.”)
- **Guided or directed orientation:** Students deal with tasks which help them to explore implicit relationships. The teacher suggests activities that enable students to recognize the properties of the new concept. The relations belonging to the context are discovered and discussed. (A teacher might ask: “What happens when you cut out and fold the rhombus along a diagonal? Along the other diagonal?”)
- **Explanation or Explication** Students formulate what they have discovered, and new terminology is introduced. They share their opinions on the relationships they have discovered in the activity. The teacher makes sure that the correct technical language is developed and used. The van Hieles thought it is more useful to learn terminology after students have had an opportunity to become familiar with the concept. (A teacher might say: “Here are the properties we have noticed and some associated terminology for the things you have discovered. Let us discuss what these mean: The diagonals lie on the lines of symmetry. There are two lines of symmetry. The opposite angles are congruent. The diagonals bisect the vertex angles.”)
- **Free orientation** Students solve more complex tasks independently. It brings them to master the network of relationships in the material. They know the properties being studied, but they need to develop understanding of relationships in various situations. This type of activity is much more open-ended. (A teacher might say: “How could you construct a rhombus given only two of its sides?” and other problems for which students have not learned a fixed procedure.)
- **Integration** Students summarize what they have learned and keep it in mind. The teacher should give to the students an overview of everything they have learned. It is important that the teacher does not present any new material during this phase, but only a summary of what has already been learned. (A teacher might say: “Here is a summary of what we have learned. Write this in your notebook and do these exercises for homework.”) Pierre van Hiele wrote: “A definition of a concept is only possible if one knows, to some extent, the thing that is to be defined.”

TABLE 2

Five phases of learning support students as they progress through levels of geometric thought.

Framework of the Van Hiele phases of learning

Phase	Descriptions
Information	Students develop vocabulary and concepts for a particular task. The teacher assesses students' interpretation/reasoning and determines how to move forward with future tasks.
Directed orientation	Students actively engage in teacher-directed tasks. They work with the developments from the previous stage to gain an understanding of them as well as the connections among them.
Explication	Students are given the opportunity to verbalize their understanding. The teacher leads the discussion.
Free orientation	Students are challenged with tasks that are more complex and discover their own ways of completing each task.
Integration	Students summarize what they have learned, creating an overview of the concept at hand.

The Van Viele Theory of Geometric Thought

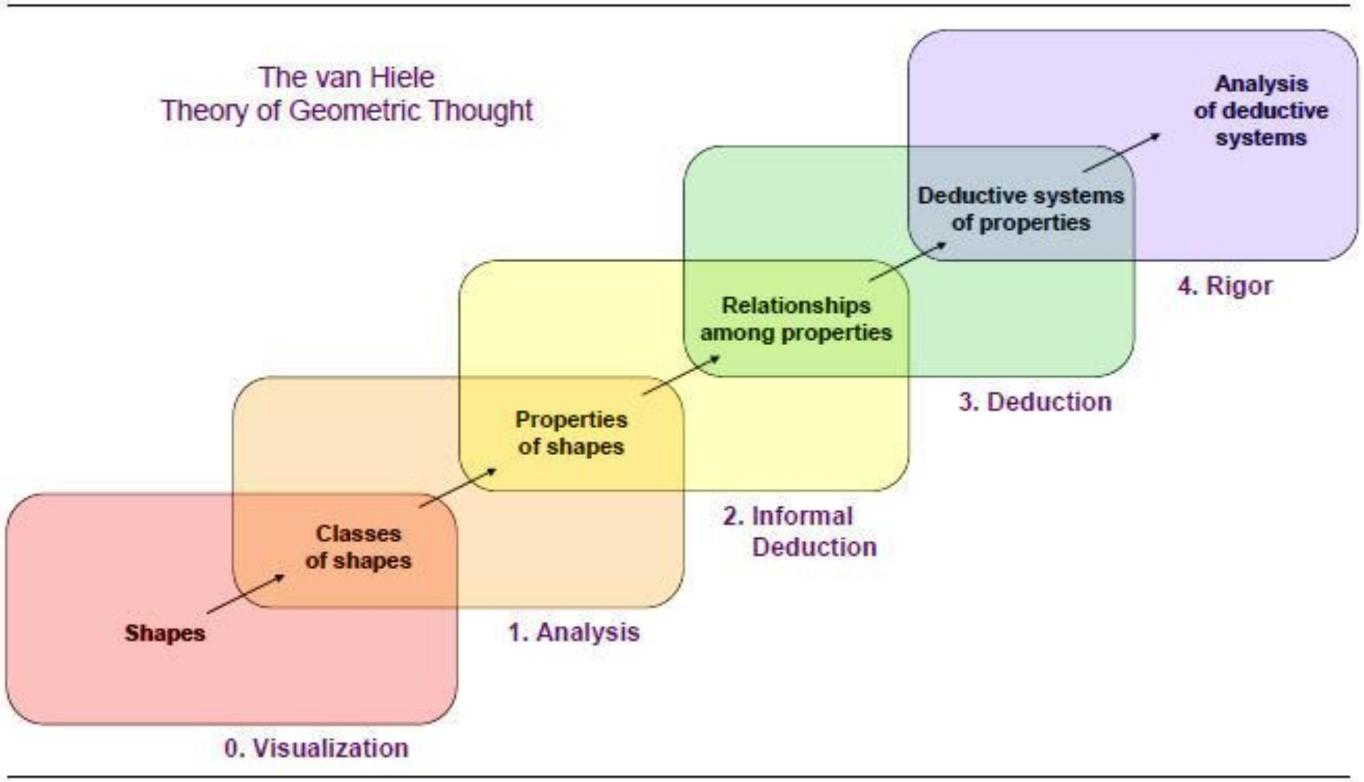

Table 1 The van Hiele model of geometric understanding describes a progression that is independent of age or grade level.

Level	Name	Description	Example	Teacher Activity
0	Visualization	See geometric shapes as a whole; do not focus on their particular attributes.	A student would identify a square but would be unable to articulate that it has four congruent sides with right angles.	Reinforce this level by encouraging students to group shapes according to their similarities.
1	Analysis	Recognize that each shape has different properties; identify the shape by that property.	A student is able to identify that a parallelogram has two pairs of parallel sides, and that if a quadrilateral has two pairs of parallel sides it is identified as a parallelogram.	Play the game "guess my rule," in which shapes that "fit" the rule are placed inside the circle and those that do not are outside the circle (see Russell and Economopoulos 2008).
2	Informal deduction	See the interrelationships between figures.	Given the definition of a rectangle as a quadrilateral with right angles, a student could identify a square as a rectangle.	Create hierarchies (i.e., organizational charts of the relationships) or Venn diagrams of quadrilaterals to show how the attributes of one shape imply or are related to the attributes of others.
3	Formal deduction	Construct proofs rather than just memorize them; see the possibility of developing a proof in more than one way.	Given three properties about a quadrilateral, a student could logically deduce which statement implies which about the quadrilateral (see **fig. 1**).	Provide situations in which students could use a variety of different angles depending on what was given (e.g., alternate interior or corresponding angles being congruent, or same-side interior angles being supplementary).
4	Rigor	Learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems.	Students should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems.	Study non-Euclidean geometries such as Taxi Cab geometry (Krause 1987).